HIGHER COURSE OUTLINE

NEW TIMETABLE

UNIT 1

1.	Properties of the Straight Line 4 weeks gradient formula distance formula gradient = tan of angle line makes with positive direction of x-axis recognise the term locus any equation of the form $ax + by + c = 0$ (a, b not both zero) represents a straight line know that the line through the point (a, b) with gradient m has equation $y - b = m(x - a)$ determine the equation of a straight line given two points on the line or one point and the gradient know that the gradients of parallel lines are equal know that the lines with non-zero gradients m_1 and m_2 are perpendicular if any only if $m_1m_2 = 1$ solve mathematical problems involving the above properties of straight lines know the concurrency properties of medians, altitudes, bisectors of the angles and perpendicular bisectors of the sides of a triangle	4 weeks exceeds the amount of time actually needed for this outcome but classes will be regularly disrupted during the last four weeks of term. Since classes change significantly once results are known it is inadvisable to progress beyond this outcome. Keep emphasising $m = \tan \theta$ – pupils keep forgetting this. Locus – this is listed in the course specification but has yet to appear in H Grade exam. This must involve problems where the gradient must be found other than by using the gradient formula. Special mention should be made of horizontal/vertical lines. Finding the equations of lines associated with triangles is very important – it is worth pointing out that the medians trisect. In the arrangements collinear points is only listed under vectors in unit three – time should be spend on the concept now and pupils shown how to prove points are collinear using gradients.

TERM 5.1

1.	Properties of the Straight Line 1 week Revision of pre-summer work	Since there will be a significant number of new pupils in each class, revision of this work is essential – pupils who unexpectedly passed Credit/Int 2 and are joining a Higher class will need extra support to get to grips with this first outcome.
2.	Functions and Graphs 3 weeks	
	know the meaning of the terms domain, range, inverse and composite function	Inverse functions are required only for obtaining the log function from the exponential function.
	know the meaning of the terms amplitude and period	
	be aware of the general features of the graphs of sin(ax + b) and cos(ax + b)	
	know that л radians = 180°	Some time should be spent on converting between degrees and radians. Revising simple trig equations at this point will prove useful later in the course.
	know the exact values of sin/cos/tan 0, $л/_6$, $л/_4$, $л/_3$, $л/_2$ rad	
	know the general features of the graphs of a^x and $\log_a x$	Pupils will find this very difficult at this stage – staff may choose to teach only enough to allow pupils to pass the unit assessment and then do a more thorough treatment when teaching logs in Unit 3.
	given the graph of $f(x)$ draw the graphs of related functions, where $f(x)$ is a simple polynomial or trig function: $-f(x)$, $f(-x)$, $f(x) + k$, $f(x + k)$, $kf(x)$, $f(kx)$, $f'(x)$ recognise the probable form of a function from its graph	Related functions where $f(x)$ is a log or exp function should also be taught but probably not until Unit 3. Graph of $f'(x)$ should be covered later in this Unit once pupils have been introduced to differential calculus. **Pupils should be able to sketch graphs of related functions where more than one transformation is required.**
	given $f(x)$ and g(x), find the composite functions $f(g(x))/g(f(x))$	The notation fog is not required.
	complete the square in a quadratic of the form $x^2 + px + q$	Questions usually ask pupils to write a given quadratic in the form (x +a)2 + b
	complete the square in a quadratic of the form $ax^2 + bx + c$	Pupils often find it easier to "work backwards"
	interpret formula and equations	Find max/min values of completed squares including those involving trig functions or of the form $a - b(cx - d)^2$

3.	Basic Differentiation 4 weeks	
	know the meaning of the terms limit, differentiable at a point, differentiate, derivative, differentiable over an interval, derived function	
	use the notation $f'(x)$ or dy/dx for a derivative	
	know that $f'(x) = \lim_{h \to 0} (f(x + h) - f(x))/h$	Differentiation from first principles is not required.
	know that the derivative of ax^n is anx^{n-1}	
	know that the derivative of the sum of two functions is the sum of the derivatives	Some revision of surds and indices may be necessary.
	know the meaning of the terms rate of change, average gradient, strictly increasing/decreasing, stationary point (value), max/min turning point (value), horizontal point of inflexion	Work should include application to acceleration/velocity and other rates of change such as rate of change of volume of a sphere with respect to the radius.
	know that $f'(a)$ is the rate of change of f at a	
	know that the gradient of the curve $y = f(x)$ at any point on the curve is the gradient of the tangent at that point and is equal to the value of $f'(x)$ at that point	Pupils will be required to find the equations of tangents to curves – this can be used as an early opportunity to revise straight line work.
	find the points on a curve at which the gradient has a given value	
	know the conditions for a function to be strictly increasing, strictly decreasing, stationary	
	find the stationary point(s) (value(s)) on a curve and determine their nature	The second derivative can be used as an alternative to a table of sign for $f'(x)$
	sketch a curve with given equation by finding stationary points and their nature, intersections with axes, behaviour of $f(x)$ for large positive/negative values of x	The use of graphic calculators should be encouraged as a check.
	determine the greatest/least values of a function on a given interval	
	solve optimisation problems using calculus	Pupils find the derivation of formula for areas/volumes etc difficult but should still be encouraged to deal with the optimisation part of any problem. Pupils frequently forget to check that value of the unknown they find does give the max/min asked for.
	give the graph of $f(x)$ draw the graph of $f'(x)$	This completes the sketching of relation functions from Outcome 2.

TERM 5.2	
4. Recurrence Relations 1/2 weeks	
know the meaning of the terms: sequence, nth term, limit as n tends to infinity	"Series" is not included but the concept has appeared in Credit exam (2003).
use the notation u_n for the nth term of a sequence	Pupils need to experience a wide variety of mathematical models of situations involving recurrence relations – the interpretation of the problem often proves more difficult than the mathematical content.
define and interpret a recurrence relation of the form $u_n+1 = au_n + b$ where a and b are constants	
know the condition for the limit of the sequence resulting form a recurrence relation to exist	Care should be taken over how pupils communicate their interpretation of the limit of a sequence.
find (where possible) and interpret the limit of the sequence resulting from a recurrence relation in a mathematical model	

UNIT ASSESSMENT

As with all unit assessments pupils should be given ample opportunity to attempt practice papers – two should be enough for most pupils. If possible, let pupils have the first of these before the October break (for homework over the holiday?) even though they are unlikely to have started the final outcome.

Aim to have this unit completed before the end of October.

Time allowed – Approximately 45 minutes. Some flexibility may be allowed to suit individual circumstances.

Re-sits – Where a pupil has not attained the standard necessary to pass a particular outcome or outcomes, they need only be reassessed on the outcomes failed. Only in exceptional circumstances will a second re-sit be allowed.

COURSE ASSESSMENT

A course assessment is available.

All pupils should sit this. It will allow pupils to appreciate the difference between the standard required in the final exam and that required for the unit assessment. The evidence generated by the course assessment can be used for appeals.

TERM 5.2 UNIT 2 1. The Equation of the Circle 2/3 weeks know that the equation of the circle centre (a,b) and radius r is $(x - a)^2 + (y - b)^2 = r^2$ know that the equation x2 + y2 + 2gx + 2*f*y + c = 0 represents a circle centre (-g, -*f*2) and radius √(g2 + *f*2 – c) provided g2 + *f*2 – c > 0 determine the equation of a circle solve mathematical problems involving the intersection of a straight line and a circle, a tangent to a circle determine whether two circles touch each other In the arrangements for Higher Maths, this is the fourth outcome of Unit 2.	The condition for the circle to "exist" needs to be emphasised. Determine the points at which a given line intersects a given circle, determine whether a given line is a tangent to a given circle, determine the point of contact of a tangent to a circle. **Determine the possible values of k if the line ax + by = k (a and b given) is a tangent to a given circle.**

2.	Quadratic Theory 2 weeks	
	know the formula for the roots of the quadratic equation $ax^2 + bx + c = 0$	The content of this block of work together with Factor/Remainder theorem form the first outcome of Unit 2.
	know that the discriminant of $ax^2 + bx + c$ is $b^2 - 4ac$	
	use the discriminant to:	
	determine whether or not the roots of a quadratic equation are, real, and, if real, whether equal or unequal	Rational/irrational roots are not included.
	find the condition that the roots of a quadratic equation are real, and, if real, whether equal or unequal	The equation may be presented in standard form or **may require some rearrangement**.
	know the condition for tangency, intersection of a straight line and a parabola/circle	
	solve quadratic inequalities	The graph of the quadratic may or **may not be given**.
	determine a quadratic equation with given roots	The formulae for the sum/product of roots are not included.

3.	Trigonometric Formulae 3 weeks	
	solve trig equations in a given interval	eg acosbx = c, 0≤x≤? eg **$3\sin^2x + 7\sin x - 6 = 0$**
	know and apply the addition formulae and double angle formulae for sin(A±B), cos(A±B), sin2A and cos2A	Standard formulae are given but pupils should be encouraged to memorise them. Trig identities form S Grade/Int 2 are worth revising at this stage. Pupils may be required to apply formulae "backwards". eg expand sin(P+2Q) eg **simplify 2sinx/2cosx/2**
	apply trig formulae in the solution of geometric problems	Applications include finding angles in 3D situations. Work should extend to problems involving compound angles.
	solve trig equations involving addition formulae and double angle formulae	In assessments solutions would be on a given interval.

TERM 5.3	
4. Factor/Remainder Theorem 2 weeks	Valuable time can be saved if this block of work can be started before the Christmas break.
use the Remainder Theorem to determine the remainder on dividing a polynomial $f(x)$ by ix – h	
determine the roots of a polynomial equation	
use the Factor Theorem to determine the factors of a polynomial	The polynomial will be at least third degree and will have at most one irreducible quadratic factor. **Cases where it has been necessary to consider values of h which are not whole numbers are rare in examinations.**
prove that an equation has a root between two given values and find that root to a required degree of accuracy	For most pupils this will be revision of S4 work.

5.	Basic Integration 3 weeks know the meaning of the terms integral, integrate, constant of integration, definite integral, area under a curve know that integration is anti-differentiation integrate functions defined by px^n and the sum or difference of such functions evaluate definite integrals determine the area bounded by the curve y = f(x), the lines x = at, xi = b and the x-axis using integration determine the area bounded by two curves solve equations of the form $^{dy}/_{dx} = f(x)$	Pupils may be expected to simplify product and rational expressions prior to integration.

UNIT ASSESSMENT

As with all unit assessments, pupils should be given ample opportunity to attempt practice papers. One should be enough for most pupils.

Aim to have the unit assessment no later than the first week in February.

Time allowed – Approximately 45 minutes. Some flexibility may be allowed to suit individual circumstances.

Re-sits – Where a pupil has not attained the standard necessary to pass a particular outcome or outcomes, they need only be reassessed on the outcomes failed. Only in exceptional circumstances will a second re-sit be allowed.

COURSE ASSESSMENT

A course assessment is available, but since the prelim exam is scheduled for the period following the INSET days in February, the course assessment should be done only if time permits.

PRELIM

The prelim will be based on Units 1 and 2 only. None of Unit 3 will be included unless the work has been covered by all classes and class teachers have been informed well in advance.

TERM 5.3 UNIT 3	
1. Vectors in Three Dimensions 2 weeks	Classes should try to make a start to this block of work before the prelim exam, but it is likely to be examined in the prelim unless classes are well ahead of schedule.
know the terms: vector, magnitude, direction, scalar multiple, position vector, unit vector, directed line segment, component, scalar (dot) product	
know the properties of vector addition and multiplication of a vector by a scalar	Pupils taking physics will already be familiar with the concept of a vector.
determine the distance between two points in three dimensional space	
know the condition for two vectors to be equal	Pupils should have an understanding of vectors in both two and three dimensions.
know and apply the fact that if u and v are vectors that can be represented by parallel lines then u = kv where k is a constant and the converse	Pupils may be required to find the coordinates or positive vector of a point which divides the join of two given points in a given ratio. The section formula may be used but is not required.
know and apply the fact that if A, B and C are collinear points such that $^{AB}/_{BC}$ = $^{m}/_{n}$ then **A**B = $^{m}/_{n}$ **BC**	
determine whether three given points are collinear	
know and apply the basis vectors i, j, k	
know the scalar product facts: **a.b** = $a_1b_1 + a_2b_2 + a_3b_3$ **a.b =\a \\b** \cosθ	
determine whether or not two vectors, in component form, are perpendicular	
use the scalar product to find the angle between two directed line segments	

2. Further Differentiation and Integration 1/2 weeks	
know and apply the rules for differentiating/integrating sinx and cosx	Pupils find it difficult to recognise the need to use the chain rule for trig function such as sin3x, $\cos^2 x$ – the introduction of appropriate brackets can often help.
know and apply the chain rule for differentiation	**Pupils will be required to differentiate functions where the derivative of the "bracketed" function is no 1.**
integrate functions defined by: $f(x) = (px + q)n$, $f(x) = p\sin(qx + r)$ and $f(x) = p\cos(qx + r)$	For any composite function which has to be integrated, the "bracketed" function will be of the form x + a **or ax + b**.
TERM 5.4	
3. Further Trigonometric Relationships 1/2 weeks	
express $a\cos\theta + b\sin\theta$ in the form $r\cos(\theta \pm a)$ or $r\sin(\theta \pm a)$	All intermediate working must be shown – the calculation of r and a, the expansion of trigonometric formulae and the equating of coefficients.
solve, by expressing in one of the forms above, equations of the form $a\cos\theta + b\sin\theta = c$	
find maximum/minimum values of expressions of the form $a\cos\theta + b\sin\theta$	
find corresponding values of θ	

3.	Logarithmic and Exponential Functions 2 weeks	
	know that $a^y = x \Leftrightarrow \log_a x = y$	Change of base is not included. The base a will normally be 10 or e. Pupils should be familiar with the notation lnx.
	now the laws of logarithms	
	simplify numerical expressions using the laws of logrithms	
	solve simple logarithmic and exponential equations	eg lnx = 2, 10x = 2.5, 3.4x = 5
	solve for a and b equations of the following forms, given two pairs of corresponding values of x and y: **logy =alogx + b** **$y = ax^b$** **$y = ab^x$**	
	use a straight line graph to confirm a relationship of the form $y = ax^b$ also $y = ab^x$	
	model mathematically situations involving the logarithmic or exponential function	**Pupils will be required to draw an appropriate graph from experimental data and reduce values of a and b such that $y = ax^b$.**

UNIT ASSESSMENT

As with all unit assessments, pupils should be given ample opportunity to attempt practice papers. One should be enough for most pupils.

Aim to have the unit assessment as soon after the Spring break as possible.

Time allowed – Approximately 45 minutes. Some flexibility may be allowed to suit individual circumstances.

Re-sits – Where a pupil has not attained the standard necessary to pass a particular outcome or outcomes, they need only be reassessed on the outcomes failed. Only in exceptional circumstances will a second re-sit be allowed.

COURSE ASSESSMENT

A course assessment is available and MUST be taken by all pupils. This assessment will usually provide the only evidence for Unit 3 in an appeal.